MEM6810 Engineering Systems Modeling and Simulation

Sino-US Global Logistics Institute
Shanghai Jiao Tong University

Spring 2024 (full-time)

Assignment 4

Due Date: June 18 (14:00)

Instruction

(a) You can answer in English or Chinese or both.
(b) Show **enough** intermediate steps.
(c) Write your answers **independently**.
(d) If you copy the solutions from somewhere, you must **indicate the source**.

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Question 1 (30 points)

We have $k > 2$ different (system) designs, and their mean performances are θ_i, $i = 1, 2, \ldots, k$. We want to select the one with the largest mean performance. Bechhofer's Procedure (Lec 9 page 19/29) can ensure that when Assumptions 1-4 (Lec 9 page 18/29) are satisfied, $\mathbb{P}\{\text{select the larget } \theta_i\} \geq 1 - \alpha$. Now we relax Assumption 3. Give a rigorous proof that, when Assumptions 1, 2, and 4 (Lec 9 page 18/29) are satisfied,

$$\mathbb{P}\left\{\left|\text{selected } \theta_i - \max_{1 \leq i \leq k} \theta_i\right| < \delta\right\} \geq 1 - \alpha.$$

Question 2 (20 points)

Explain why the Paulson's Procedure (Lec 9 page 24/29), under Assumptions 1-3 (Lec 9 page 18/29) and common known variance assumption, will stop almost surely (i.e., with probability one). Try to be as rigorous as possible.

Question 3 (50 points)

Consider the simulation optimization problem,

$$\min_{\boldsymbol{x} \in \mathcal{X}} \; g(\boldsymbol{x}),$$

where $g(\boldsymbol{x}) \coloneqq \mathbb{E}[G(\boldsymbol{x})]$ and $G(\boldsymbol{x})$ is the output of a simulation replication conducted at $\boldsymbol{x}$. Let $\boldsymbol{x}^*$ be a global optimal solution. Grid search is often used to find a global optimal solution to the problem. It first chooses m grid points, $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_m$, in $\mathcal{X}$.

It then takes r i.i.d. observations from each of the m grid points and calculates the sample means, $\bar{G}(\boldsymbol{x}_1), \bar{G}(\boldsymbol{x}_2), \cdots, \bar{G}(\boldsymbol{x}_m)$. Let

$$\hat{\boldsymbol{x}}_m^* = \arg\min\{\bar{G}(\boldsymbol{x}_1), \bar{G}(\boldsymbol{x}_2), \cdots, \bar{G}(\boldsymbol{x}_m)\} \text{ and } \boldsymbol{x}_m^* = \arg\min\{g(\boldsymbol{x}_1), g(\boldsymbol{x}_2), \cdots, g(\boldsymbol{x}_m)\}.$$

Suppose that the grid points are chosen such that $g(\boldsymbol{x}_m^*) \to g(\boldsymbol{x}^*)$ as $m \to \infty$. (How to ensure the above condition is of course an important question in practice. Here we simply assume we can do it.) We further assume that $\sup_{\boldsymbol{x}\in\mathcal{X}} \mathrm{Var}[G(\boldsymbol{x})] = \sigma^2 < \infty$. In order to ensure that $g(\hat{\boldsymbol{x}}_m^*) \to g(\boldsymbol{x}^*)$ almost surely as $m \to \infty$, r and m need to satisfy some relationship. Prove that, if r will increase when m increases (that is to say $r = r(m)$ is an increasing function on m) and

$$\sum_{m=1}^{\infty} \frac{m}{r(m)} < \infty,$$

then the above almost sure convergence holds.

Hint: You may need to use the following fact: $|\min_{i=1,\dots,k}\{a_i\} - \min_{i=1,\dots,k}\{b_i\}| \le \max_{i=1,\dots,k}\{|a_i - b_i|\}$, for any given $\{a_1, \dots, a_k\}$ and $\{b_1, \dots, b_k\}$. You can find some idea from [L. Jeff Hong, Barry L. Nelson (2006). Discrete optimization via simulation using COMPASS. *Operations Research* **54**(1):115-129. https://doi.org/10.1287/opre.1050.0237]